A Note on Search and Assortative Matching in Wealth

PRELIMINARY AND INCOMPLETE, PLEASE DO NOT CITE.

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Abstract

Empirically, we often observe that the characteristics of both partners, in particular wealth, in a match are positively correlated. In many models of investment in descendants and intergenerational wealth mobility, this point is often taken as the starting premise of the model. But can we explain *why* people match assortatively in wealth? In a world without frictions, with non-transferable utility, the assortativeness is no surprise: each person is ranked in order of quality, and is matched with a person of the same rank of that quality, when utility is increasing in the quality of both sides (Becker 1973,1974): positive assortative matching (PAM). However, when frictions are introduced, the conditions for positive assortative matching are not as straightforward. In the literature it is derived that the condition for PAM is a utility function exhibiting log-supermodularity. This requirement seems restrictive; it might appear difficult to reconcile log-supermodularity with the empirically observed positive assortative matching in wealth, when marginal utility of additional wealth is declining in one's own wealth level. Here, we show that in fact it is extremely simple and straightforward, *without any log-supermodularity assumption*, to get positive assortative matching in wealth, while marginal utility of wealth is decreasing. The condition for this is a utility function that exhibits decreasing

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absolute risk aversion, and a utility flow of being single that depends positively on one's own wealth.

1 Introduction

Empirically, the wealth of both partners in a marriage exhibits significantly correlation. We see that matches are sorted positively in education and wages (Becker 1973, Lam 1988), factors that clearly in part determine one's wealth. However, even within groupings of people with the same jobs and education, we observe indicators that matching is positive assortative in wealth. Over the entire population, Charles, Hurst and Ren (2005) have found from PSID data that on average the correlation between the parent's wealth of the husband with the one on the wife's side is 0.42, and the assortative tendency remains within much more homogenous subgroups. That this particular correlation is this strong on top of things that are prime determinants of wealth like education and wages, tells us that overall wealth is a very important factor in determining who matches who.

Indeed, there is a literature that takes the assortative aspect of marriage matching as given, and looks at the resulting incentives for investment, and in particular at investment in descendants. The argument roughly goes like this: if parents invest a lot in children than they do not only get the (labor) market return to their investment, but also have their child marry rich (Cole, Mailath and Postlewaite 1992, Siow and Peters 2002; Fernandez, Guner and Knowles 2005). If some of the wealth goes into household public goods, then there is clearly an additional, 'matching' gain of wealth that individuals take into account when making economic decisions. Given a theory of matching in wealth, we can apply it to issues such as intergenerational mobility within the wealth distribution: a society that is more assortative in wealth is also a much more stratified wealth distribution. On the other hand, the degree towards which a society has matching assortative in wealth, also affects the incentives to accumulate wealth, be entrepreneurial, et cetera, exactly because of the 'matching' gains of more wealth.

To understand what assortative matching does, we need to understand why matching is assortative in wealth in the first place. If matching is frictionless, this is very simple to understand (and therefore often left out unspoken): since everybody wants the highest wealth he or she can get, the person with the highest wealth is never refused, and chooses the person with the highest wealth on the other side. Of the remaining persons, the now highest person chooses the remaining highest person on the other side, etc. Thus, matching is straightforwardly based on rank.

However, it is argued in many places (most prominent among them, Burdett and Coles, 1997, 1998; Mortensen 1985, ...), that the models with meeting frictions (bilateral search models) capture particularly relevant aspects of the marriage market. Casual empirics strongly suggest that we do not meet all possible marriage candidates at once, we meet them mostly one at a time; after observing a person's characteristics we decide to invest in the relationship or to start looking for the next meeting again, which can take time to realize. This is captured nicely in the setup of the search models. The question then is: does the marriage model with frictions yield assortative matching in wealth?

This, however, is not immediately clear from the setup anymore. When frictions are introduced, matching is not straightforwardly based on rank. A person will have to choose to accept a randomly drawn person or to wait for the next random draw, next period. Individuals with different wealth might look differently to the trade-off of time versus immediate 'consumption' of a match. It could be the case that wealthier persons are more impatient, and match sooner. But this has equilibrium effects, as less wealthy individuals on the other side now face extended possibilities. They might start to refuse the type they were matched with in the frictionless case, just because they are holding out to meet an impatient rich person. Since rich persons disappear quickly out of the distribution it is too costly for the impatient rich person to wait for a match with another rich person. This possible equilibrium reasoning is indicative of the possibilities with frictions. Thus, it becomes interesting to derive the conditions under which assortative matching will still result[1].

Recently, a number of papers have investigated the conditions for positive assortative matching in markets with frictions. In the case most relevant to us, of frictional matching with nontransferable utility, Smith (1997) has argued that utility needs to be logsupermodular (not necessarily supermodular) for PAM to arise,

[1]Note that in this context, assortative matching is in a 'set'-sense: a person will accept a range of other persons, and a person with more wealth will accept less people at the bottom, and be accepted more. Nevertheless some of the people could be accepted by both persons.

in case of non-transferable utility, the case that we focus on here.[2] [3]Smith's logsupermodularity condition at first sight might seem strong; it might appear hard to reconcile a standard utility specification, additive in wealth, with this utility specification. Logsupermodularity supposes that the relative benefit of matching up is increasing in type. In other words, a poor person gains 10% from marry a bit richer person, whereas a rich person gains 20% from marrying the same person (relative to matching with the same baseline person). This seems to contradict the standard economic utility specification that marginal utilities are decreasing (in this context both private and public goods). As we will see, a crucial -yet not always noted- driving force of the need for log-supermodularity is that the utility value of being unmatched is zero. This makes that rich people lose relatively much more from being unmatched; to make the rich patient, this can only be offset by sharply increasing relative gains for matches with higher types.

Put differently, with frictions, the utility function really plays two roles: it defines the complementarities between types, but it also tells us about the valuation of the gamble that an individual is constantly considering: "Should I stick with what I have now, or should I draw randomly again in the next period?" How individuals value the risk plays thus an important role. In particular, one can imagine that, if high types are very risk-averse, and low types are much more risk-loving, we will not necessarily get PAM. High types will want to match immediately, whereas low types will want to wait for a high type to come along (and therefore won't match each other). This effect could offset complementarities. In the previous literature, this effect was mostly referred to as 'the value of time', a term that could obscure the role of risk preference in this.

The question of this paper is simple. When we restrict the risk-aversion of the agents to rule out such a behavior, do we get positive assortative matching under

[2]We focus here on the NTU case, because it is very difficult to bring risk-aversion into a TU framework, without loosing the one-for-one transferability of utility. Moreover, NTU relates to the case that an economically significant part of the consumption in households are be local public goods, while explicit contracting on contributions towards it is impossible.

[3]Related to this, in different setups, Shimer and Smith (2000) have derived conditions for matching to be positively assortative in the TU case with frictions and discounting, Atakan (2006) has found that utility has to supermodular when there is a fixed cost of sampling again; likewise Chade (2001) has found that supermodularity suffices for the case of fixed costs in the NTU game.

the frictionless assumptions? What assumptions on the risk-aversion parameters do we need to make?

2 Simple Setup and Acceptance Decisions

We use a simplified setup of the standard bilateral search models (as in e.g. Burdett and Coles 1997, Eeckhout 1999, and Smith 2002), but allow for different utility functions (or production functions, as Smith calls them) and, importantly in specification 1, *for nonzero, heterogeneous, utility of being unmatched*. In short we have the following:

◇ N single women, N single men

◇ Each woman has wealth ω_w according to the continuous distribution $F_w(\omega_w)$, each man according to continuous$F_m(\omega_m)$, both with bounded interval support, on an interval $\Omega_i = [\underline{\omega_i}, \bar{\omega_i}]$

◇ Standard two-sided matching, *random* meetings.

◇ α arrival rate of potential partners

◇ Marrying singles are replaced by clones. (This is a major simplification, with great convenience. However, none of the results are affected qualitatively. Alternatively, at the cost of analytical difficulty, we can do this in a Burdett and Coles (1997) steady state analysis, which does not change our results qualitatively, for the same reasons they mention.)

◇ infinitely lived agents

◇ look only at steady state; even with clones is it necessary to confine looking at the steady state. This means that the overall distribution of wealth will not change from meeting to meeting

◇ stationary strategy

◇ there is no on-the-job search, no endogenous separation and polygamy is ruled out as well.

Importantly, we use two different specifications of utility:

`Specification 1:`

Utility is additive in the two agents' wealth:

$$u(w_m + w_f);$$

it exhibits decreasing absolute risk aversion. The utility of not being matched, $u_0(w_i)$, is $u(w_i)$. Thus a rich man derives more utility from being single than a poor man. We assume that $u(\underline{\omega_i}) \geq 0$.

`Specification 2:`

Utility is log-supermodular in both types $u(w_f, w_m)$. The outside option $u_0(w_i)$ is 0 for every individual i.

As is shown in the literature (see Burdett and Coles 1997, and Eeckhout 1999), this boils down to the following value functions:

$$V_0(w_m) = \frac{1}{1+r}\left(u_0(w_m) + \alpha \int_{\Omega_f(w_m)} \max\{V_1(w_m, w_f), V_0(w_m)\} + (1-\alpha)V_0(w_m)\right), \tag{1}$$

where $V_0(w_m)$ denotes the value function of an unmatched man with wealth w_m, and $V_1(w_m, w_f)$ of the same man matched with a woman with wealth w_f. The set $\Omega_f(w_m)$ consist of all women would want to match with this man of type w_m. (For clarity, we formulate the problem mainly from the view point of the men[4].) Given that we assumed that matches last forever, we have the following value function for a man who is actually matched with a woman with wealth w_f is

$$rV_1(w_m, w_f) = u(w_m, w_f), \tag{2}$$

we have that $V_1(w_m, w_f)$ is increasing in w_f. Hence the optimal policy is a *reservation wealth policy.* Thus it makes sense to characterize a person's decisions as a function mapping one's type into the lowest type that a person accepts, given the acceptance decisions on the other side. Let us define the **accepting function**, a mapping $\mathcal{A}_m(w_m, w_f)$ that exactly does this: given that one is of type w_m, is accepted by women of type w_f and lower, it gives the reservation wealth above which all women are accepted. If we were to know this function, under appropriate assumptions, we can derive the equilibrium straightforwardly. This is done in the next section.

[4]To avoid gender discrimination, this choice was based entirely on a coin toss.

3 Equilibrium given Accepting Functions

We split up the derivation of the equilibrium in two steps. First, we focus on the derivation of the equilibrium given the accepting functions of all types on both sides. Of course, to have optimization, it needs to be the case that the accepting functions are given by the optimal decisions of each individual. We discuss this in the next session, for now we take the accepting function as given to derive the equilibrium.

The purpose of this section thus is to show that given increasing accepting functions we have an equilibrium with positive assortative matching. Then, in the next section, it suffices to show that both specifications lead to increasing accepting functions. Let us now spell out explicitly what we mean by increasing acceptance functions; keeping in mind that the assumption here will be derived to be the result of the optimal behavior, given the two specifications:

Assumption 1 *The accepting function, holding the highest accepting person on the other side constant, is increasing in one's wealth. I.e. if $w_i' > w_i$, then $\mathcal{A}_i(w_i', w_j) > \mathcal{A}_i(w_i, w_j)$. Moreover, it is also increasing in its second argument, holding the first constant. I.e. if $w_j' > w_j$, then $\mathcal{A}_i(w_i, w_j') > \mathcal{A}_i(w_i, w_j)$.*

This implies that a person is accepted by everyone up to a certain cutoff wealth. Thus, it makes sense that the accepting decision only depends on one's own type and the highest type accepting on the other side, as is given by the accepting function. Let us define one more function: the equilibrium accept*ed* function $\mathfrak{a}_m(w_m)$, which assigns to each man w_m the highest ranken woman that is still accepting him; and vice versa. Now, we can define an equilibrium based on accepting functions.

Definition 1 (*Partial* equilibrium given accepting function) *Let $\overline{w}_f$ be highest type of woman, $\overline{w}_m$ the most wealthy man. An equilibrium of the matching game given the accepting functions, is a tuple of functions*

$$\{\mathcal{A}_m(w_m, w_f), \mathcal{A}_f(w_f, w_m), \mathfrak{a}_m(w_m), \mathfrak{a}_f(w_f)\},$$

such that we have:

I. ***Equilibrium Accepting Decisions*** *For a man with wealth w_m, given the equilibrium accepted function, the accepting decision is to accept every woman of wealth weakly greater than $w_f = \mathcal{A}_m(w_m, \mathfrak{a}_m(w_m))$.*

II. ***Rationally Expected Acceptance*** *The accepted function of a man with type* w_m *is given by* $\mathfrak{a}_m(w_m) = \sup\{\Omega_f | \mathcal{A}_f(w_f, \mathfrak{a}_w(w_f)) = w_m\}$ *for all men* $w_m \leq \mathcal{A}_f(\overline{w}_f, \overline{w}_m)$. *For men with* $w_m > \mathcal{A}_f(\overline{w}_f, \overline{w}_m), \mathfrak{a}_m(w_m) = \overline{w}_f$. *Likewise, for women on the other side.*

We can define the equilibrium accepting function $\mathcal{A}_i^{eq}(w_i) = \mathcal{A}_i(w_i, \mathfrak{a}_i(w_i))$. Let us now proceed by deriving the equilibrium accepted and accepting function, and its implications. A first step in this is to spell out explicitly which properties the regular accepting function can have:

Definition 2 *An accepting function* $\mathcal{A}_i(w_i, w_j)$ *is called well-behaved if it it is continuous in both arguments, and for all* $w_j > \underline{w}_j + \delta$, $\delta > 0$ *we can find an* $\varepsilon > 0$, *such that* $\mathcal{A}_i(w_i, w_j) < w_j - \varepsilon$, *for all* $w_j > \underline{w}_j$.

The last part is important for finding the equilibrium, and is shown to generically hold for this setup with frictions. We also need to be clear on what positive assortative matching means in an environment with frictions.

Definition 3 *Positive assortative matching occurs when given* w_i, w_i', *and a* w_j *such that, when we have a* $w_i < w_i'' < w_i'$ *and* $\mathcal{A}_i^{eq}(w_i) \leq w_j \leq \mathfrak{a}_i(w_i)$ *and* $\mathcal{A}_i^{eq}(w_i') \leq w_j \leq \mathfrak{a}_i(w_i')$, *it has to be the case that* $\mathcal{A}_i^{eq}(w_i'') \leq w_j \leq \mathfrak{a}_i(w_i'')$. *We speak off strict assortative matching when the weak lower inequality sign is replaced by a strict one.*

It is easy to see that having increasing $\mathcal{A}_f^{eq}, \mathcal{A}_m^{eq}, \mathfrak{a}_f, \mathfrak{a}_m$ implies positive assortative matching.

Now, the definition of the partial equilibrium might already have hinted at an iterative procedure to find the partial equilibrium. Indeed, given well-behaved accepting functions, we can construct a unique equilibrium (following Eeckhout 1999).

Proposition 1 *Given that the acceptance function is well-behaved, and increasing over in its own type (holding the highest accepting type on the other side constant), equilibrium exists, is unique*[5]*, and matching will be positively assortative.*

Proof By construction.

Step 1. Start of by finding $w_i^1 = \mathcal{A}_j(\overline{w}_j, \overline{w}_i)$, for $i, j = m, f; i \neq j$.

[5] This is the only point for which the cloning assumption is important.

STEP 2.1 Then, for $w_i > w_i^1$, we will have $\mathfrak{a}_i(w_i) = \overline{w}_j$. We can calculate $\mathcal{A}_i^{eq}(w_i)$, between w_i^1 and $\overline{w}_i$. On this interval $\mathcal{A}^{eq}$ is continuous and increasing. Do this for $i = m, f$

STEP 2.2 If $\mathcal{A}^{eq}$ is strictly increasing between w_i^1 and $\overline{w}_i$, we can invert it, and derive $\mathfrak{a}_j(w_j) = (\mathcal{A}_i^{eq})^{-1}(w_i)$. If not, it must have been constant on some interval: we can still derive $\mathfrak{a}_j(w_j)$ as specified in the equilibrium definition. In this case, w_j', the supremum of the types in the interval with constant reservation wealth w_i', will be mapped into: $\mathfrak{a}_i(w_i') = w_j'$. Do this for both $i = m, f$. It is easy to show that $\mathfrak{a}_i(w_i)$ is increasing in type by the increasingness of $\mathcal{A}_j^{eq}$.

STEP 3.1 In case the acceptance decision was constant for both $i, j = m, f; i \neq j$: if $w_i^1 = \mathcal{A}^{eq}(w_j^1)$ for $i, j = m, f; i \neq j$, delete $[w_i^1, \overline{w}_i]$, for both $i = m, f$ from the distribution and start over at step 1. with the updated distribution.

In case $w_i^1 > \mathcal{A}^{eq}(w_j^1)$, for some $i = f, m$ we have gained additional information. Define $w_i^2 = \mathcal{A}^{eq}(w_j^1)$. Then, in particular, we have found $\mathfrak{a}_i(w_i)$ for every $w_i$between w_i^2 and w_i^1, since $\mathcal{A}_j$ is well-defined and continuous for every $w_j^1 \leq w_j \leq \overline{w}_j$. Given the increasingness of $\mathcal{A}_j$, we can show that $\mathfrak{a}_i$ is also increasing. Then, we can derive $\mathcal{A}_i^{eq}(w_i)$ between $w_i^2 \leq w_i < w_i^1$. Do this for both $i = m, f$.

STEP 3.2 Then, as in step 2.2, derive $\mathfrak{a}$ from the $\mathcal{A}$ functions.

Keep repeating the derivation of $\mathfrak{a}$ from $\mathcal{A}$ and the derivation of $\mathcal{A}$ from $\mathfrak{a}$.

Given the accepting functions are well-behaved, we can continue this procedure to find the acceptance decision of any $w_l > 0, l = m, f$ in finite iterations. To see why, given any w_j, w_i, the currently lowest found reservation type on both sides, we always have can find $\mathfrak{a}_i(w_i)$ for $w_j - \varepsilon \leq w_i \leq w_j$. Thus, this procedure guarantees that we find the equilibrium accepting and accepted function for $[w_l, \overline{w_i}]$, $i = m, f$, with $w_l > 0$, in finite iterations.

The result of the iterative procedure will be $\mathcal{A}_m^{eq}, \mathcal{A}_f^{eq}$ and $\mathfrak{a}_f, \mathfrak{a}_m$ which are both increasing in type. ■

4 Matching in absence of allocation decisions

In this section, we consider the case where both partners cannot make any allocation decision after matching between public and private good consumption. In the simplest case is when simply all purchased goods are consumed jointly. Alternatively, we could take a reduced form approach, and just assume properties of consumption sharing as a function of income. An example would be the case that constant ratios of income spent on the public good. Of course, this is nothing but a stepping stone for the more economic theory where matching decisions are made with the equilibrium allocation choices of the potential partner in mind. We attempt to cover that case in the next section.

4.1 All consumption is a local public good

In the section above we have shown that, given a well-behaved, and increasing accepting function, equilibrium will exhibit positively assortative matching. In this section we want to show that both continuous logsupermodular utility functions (with outside option equal to zero), and continuous utility functions with decreasing risk-aversion and outside options increasing in type lead to a positive assortative matching.

Proposition 2 *Given identical accepted types, w_f, the accepting function is increasing and continuous, both in its own type, and in the type on the other side, for both specifications.*

Proof From the specification, we have to make sure first that what is accepting function is doing in fact exists. Thus, we have to make sure that an optimal response to a given connected set of accepting types on the other sides exists (and is stationary). Then we check if the solution to the individual's decision problem, is a well-behaved function, as defined above, and is indeed increasing in each of both arguments. This done a sequence of lemmas. First let us repeat the decision problem of the worker with wealth w_m, given acceptance by a connected set of women with highest type $\tilde{w}_f$, and a value of being unmatched (used recursively as next period's continuation value in case nobody good enough comes along, given the stationary acceptance decisions) $V_0(w_m)$ given by[6]:

$$\frac{1}{1+r}\left(u_0(w_m)+\alpha\int_{\overline{w}_f}^{\tilde{w}_f}\max\{V_1(w_m,w_f),V_0(w_m)\}dF_w(w_f)+(1-\alpha)V_0(w_m)\right),$$

[6]For the derivation, see e.g. Kenneth Burdett and Melvyn Coles 1997, and Burdett and Coles 1999).

The next lemma tells us how we can find the value of being unmatched for a man with wealth w_m.

Lemma 1 *For a given w_m, $V_0(w_m)$ specifies a fix point of the mapping*

$$T(V) = \frac{1}{1+r}\left(u_0(w_m) + \alpha \int_{\tilde{w}_f}^{\tilde{w}_f} \max\{V_1(w_m, w_f), V\} dF(w_f) + (1-\alpha)V\right). \quad (3)$$

The mapping in (3) *is a contraction in $\mathbb{R}$. Moreover, the sequence V, $T(V)$, $T(T(V))$, $\ldots$ converges monotonically to the fixpoint V^*. Denote the fixpoint for a given wealth w_m by $V^*(w_m)$.*

Take $V, V' \in \mathbb{R}$. Then $|T(V) - T(V')| < \frac{1}{1+r}|V - V'|$. To show monotone convergence, consider $V^* > V$, we want to show that $V^* > T(V) > V$. Similarly, for $V > V^*$. (need to expand) $\square$

Lemma 2 *A person with a higher wealth, is more picky given the same acceptances on the other side. A person with who is accepted by more people (thus, also by better some 'better' people) is more picky.*

The proof is not hard, but it does we have to introduce some additional notation that might work a bit confusing. Take a baseline person with wealth w_m, and a person with higher wealth w'_m (thus, $w'_m > w_m$). The optimal policy is a reservation type policy, where the reservation type of w_m given $V^*(w_m)$ (the fixpoint value) will be denoted as $w^*_f(w_m)$; thus $V^*(w_m) = u(w_m, w^*_f(w_m))/r$. Now let $\tilde{V}(w'_m, w^*_f)$ or $\tilde{V}$ for short, denote the value that the richer type w'_m gets when it also has the reservation type of the poorer guy, $w^*_f(w_m)$ as his possibly suboptimal reservation type. We want to show that indeed w_m is more picky, i.e. his reservation type is higher than $w^*_f(w_m)$. From lemma 1 and the monotonicity of the utility function, it is enough to show that $T\tilde{V} > \tilde{V}$.

$$T\tilde{V} = \frac{1}{1+r}\left(u_0(w'_m) + \alpha\left[\int_{w^*_f}^{\overline{w}_f} \max\{V_1(w'_m, w_f), \tilde{V}\}\right] + (1-\alpha)\tilde{V}\right).$$

Using the fact that $\tilde{V} = u(w'_m + w^*_f)/r$, we have

$$T\tilde{V} = \frac{1}{1+r}\Bigg(u_0(w'_m) + \alpha\Big[\int_{\underline{w}_f}^{\tilde{w}_f} \max\{\frac{u(w'_m + w_f)}{r}, \frac{u(w'_m + w^*_f)}{r}\}\Big] + (1-\alpha)\frac{u(w'_m + w^*_f)}{r}\Bigg). \quad (4)$$

Rewriting the RHS of this equation again, we get

$$\frac{1}{r}\left(\frac{ru_0(w'_m)}{1+r}+\frac{\alpha}{1+r}\int_{w^*_f(w_m)}^{\tilde{w}_f}u(w'_m+w_f)dF(w_f)\right.$$
$$\left.+\frac{\big(1-\alpha+\alpha F(w^*_f(w_m))\big)}{1+r}u(w'_m+w^*_f)\right); \tag{5}$$

but this is a gamble with a payout of 0, with probability $r/(1+r)$, a payout of $w^*_f(w_m)$, with probability $\big(1-\alpha+\alpha F(w^*_f(w_m))\big)/(1+r)$, and a payout of $w_f > w^*_f(w_m)$, with density $(\alpha f(w_f))/(1+r)$. It is easy to check that all probabilities add up to one. The certainty value of this gamble, for type w_m is $w^*_f(w_m)$, by construction (given the fixed point). Since the utility exhibits decreasing absolute risk aversion with respect to money (here: female type) gambles, it must be that certainty equivalent of this gamble for w'_m is higher, but this value coincides with $T\tilde{V}$. Thus, $T\tilde{V} > \tilde{V}$. □

Above, we derived the solution to the individual's decision problem, and showed that acceptances are indeed increasing in own type and in the highest accepting type on the other side. Now, we want to make sure that the function $\mathcal{A}$ that is defined by the solutions to the problem above, satisfies certain regularity principles.

Lemma 3 *The resulting function is continuous.*

Lemma 4 *There is always a positive measure of agents accepted for every wealth above zero.*

■

The intuition of the proof is very straightforward. For a given acceptance set of women, each man effectively faces the same gamble. A man with a lower type, has a lower certainty equivalent of any gamble, because of the DARA utility, and is therefore earlier satisfied with what he has. I.e. when the wealth of the woman he currently met is above the certainty equivalent of his equilibrium gamble (given acceptance sets), then he prefers to keep her. A higher type, who has a higher certainty equivalent might want to gamble again (i.e. reject her, and randomly draw a new woman in the next meeting, again).

Thus, we have showed the following result:

Theorem 1 *Given utility is additive in wealth, exhibits decreasing absolute risk aversion, and in case of a match failure one gets utility from one's own wealth; given matching with frictions (discounting), the equilibrium matching is strictly positive assortative in types.*

Proof Follows from combining proposition 1 and 2. ■

4.2 Reduced Form Cases

We can consider a more general sharing rule of income: a constant part of income is consumed jointly. The rest is consumed in private goods. Alternatively, the interpretation is one of altruism inside the household: a husband cares about his wife's consumption, but not as much as about his own consumption. This could be captured in reduced form by a function $f(w_m, w_f)$ which tells us how much of a wife's consumption with wealth w_f is shared with a man with wealth w_m. Likewise, we have a similar sharing function of the husband's consumption for the wife.

The crucial lemma in determining the matching is lemma 2. It can be easily extended to different reduced-form sharing rules. Utility of a woman with wealth w_f matching a man with wealth w_m, is then given by $u(w_f + f(w_f, w_m)$.

Lemma 5 *Consider the following sharing rules: a)* $f(w_f, w_m) = f(w_m)$*; and b)* $f(w_f, w_m)$ *such that* $\partial f/\partial w_f < 0$, $\partial f/\partial w_m > 0$ *but* $\partial^2 f/\partial w_f \partial w_m < 0$(!?!)

Part a) is straightforward. The logic of lemma 2 implies immediately, as everybody faces the same money gambles again. Part b) is trickier. What the proposition says is that the poor get more from marrying the rich, but this difference is shrinking as the partners get more rich. Thus a very rich wife supports a poor man and a man who is a little bit better off almost identically, whereas a moderately rich wife might really support a poor man, but not so much the bit better off man. How to prove this? Note that b) effectively makes rich people get more out of rich people, relatively. Just like logsupermodularity. How to show this: Jensen's inequality. Concavity?

4.3 Matching in classes and an Equivalence Result

Let us now return to the case where all consumption is joint. With decreasing absolute risk aversion, we derived that matching was strictly positive assortative. As demonstrated by e.g. Smith (2002), and Eeckhout (1999), the resulting matching set (i.e. those that a type accepted, *and* she is accepted by) that is different for every type (see graph?). However, for the case of logmodular utility they derive that matching is positive assortative in classes. Thus, each person in one class has the same matching set as every other person in his class. The condition to get this kind of matching in wealth in our setup is a utility matching that exhibits constant absolute risk aversion.

Proposition 3 *Given utility is additive in wealth, exhibits constant absolute risk aversion, and in case of a match failure one gets utility from one's own wealth; given matching with frictions (discounting), the equilibrium matching is perfect segregation in classes. A logmodular utility with a zero utility value of being unmatched would give the same result.*

This follows from lemma 1-4, with lemma 2 updated, in lemma **??**, below.

...